frequently composed with אח, and עם (which is perhaps the abridged name of עמון ?). The next chapter, which has for subject totemism, is well treated by Mr. Jacobs in his book, *Studies in Biblical Archaeology* (1894). iv. Chapter VIII has for object the representation of the Godhead by fire, by dreams, by clouds, in the compounds of חזה, מלאך, פנים, פסל, and סמל (Semele ?). The next chapter treats of the usages of the cult. This is implied, according to our author, in words like עמרי from עמר, חני, שבתי, חרמה; these derivations are very weak. The last two chapters, which treat of the patriarchs and of the cults of the separate tribes, are rather weak, and seem too hurriedly done, the author seeming tired of his subject In general, the monograph is very dry and indigestible even for specialists. Indeed, how could the religion of the Semites be properly treated in seventy-five pages? We hope that our author will take his time for a second edition, and above all be kind enough to give an index for the thousand or more names of various kinds treated in it.

We may mention that Mr. G. Buchanan Gray, M.A., of Mansfield College, Oxford, has in the press a similar work of some 300 pages, entitled *Studies in Hebrew Proper Names*, which we hope will add materially to the knowledge of the subject.

A. N.

ABRAHAM IBN EZRA'S ARITHMETIC.

Sefer ha-mispar. Das Buch der Zahl des R. Abraham ibn Ezra, herausgegeben, ins Deutsche übersetzt u. erläutert von Dr. M. SILBERBERG. (Frankfurt a. M.: S. Kaufmann. 1895.)

MATHEMATICS formed a favourite study of Ibn Ezra; wherever opportunity is given in his writings he introduces Geometry, Arithmetic or Astronomy. In the curriculum of studies which he recommends to his pupil in his *Yesod Mora* or "Foundation of the Fear of the Lord," the mathematical sciences occupy a prominent place. The numbers seem to have frequently engaged his attention, and three pamphlets were written by him on numbers, from three different aspects. *Yesod ha-mispar*, "Foundation of the numerals," is a treatise on the grammatical peculiarities of the numerals; whilst, in the *Sefer ha-eḥad*, "The Book on the Unit," the author discusses the theory of the numbers. The *Sefer ha-mispar* was probably intended to be a guide in Elementary Arithmetic. The dedicatory lines which in some

MSS. and in the present edition precede the *Sefer ha-mispar*, are out of place here. The author says, Behold here is a book faithfully copied; in it you find of every number its properties; it is written by Meir's son for Meir, who is young in years but great in understanding. It is in the book *Sefer ha-eḥad* that the properties of each number are described, and not in the *Sefer ha-mispar*. The editor wrongly translates here *mispar* by "problem or sum," and *t'chunah*, by "solution," and is, like some copyists, misled to connect these lines with the *Sefer ha-mispar*. It is also possible that *Sefer ha-mispar* is not at all the original title of this book.

Who is this Meir to whom these lines are addressed? Mr. Halberstamm, in *Sefer ha-ibbur*, believes that he is the same Meir whom Maimonides mentions in his letter to Rabbi Samuel Ibn Tibbon as a pupil of Ibn Ezra. The book opens with a parallelism between the Universe and the numbers; there we have nine spheres and a being that is the beginning and source of all the spheres, and at the same time separate and different from the spheres. Similarly there are nine numbers, and a unit that is the foundation of all numbers but is itself no number. There are only nine spheres, and only nine numbers; ten is again one in the second series, and so hundred in the third and so on. The nine numbers are represented by the first nine letters of the Hebrew alphabet, and their absence whether it be in the units, or tens, &c. is indicated, as in the ordinary systems, by a circle. Ibn Ezra retains also the usual order, and counts from right to left, the first place being assigned to the units, the next to the tens, &c., e.g. אה = 15; כ = 20; אבכ = 3021. By means of writing the nine figures symmetrically in a circular form he shows a certain peculiarity of the multiples of nine, viz. that each multiple from two to five consists of two opposite figures, the one giving the units the other the tens, and from six to nine, the multiples contain the same figures in reversed order as regards the units and tens, e.g. $2 \times 9 = 18$ and $9 \times 9 = 81$. The same remark, in the same context, is found in Ibn Ezra's Comm. on Ex. iii. 14, and in other writings of Ibn Ezra. He seems to see in this peculiarity a proof that nine figures suffice for all purposes, since they suffice to express all multiples of nine, which is the greatest number. The four species are not arranged in the same order as we have them in our Arithmetic books. Multiplication and division precede here addition and subtraction. This is certainly unmethodical, because multiplication is not completed without addition, nor is division worked out without subtraction. Ibn Ezra no doubt followed in this respect the arrangement generally adopted in Arithmetic books of his time. It may be that the reason for this strange arrangement is to be found in the fact that the arithmetical problems

which are foremost in practical life, are multiplication and division. Numerous problems are solved by the author in illustration of the method taught by him. It is worth noticing that whilst modern Arithmetic books are rich in problems about interest, the Arithmetic written by a Jew for Jews does not contain one single sum dealing with interest. His method of multiplication and division is the same as our method. The difference exists only in the way in which the successive steps of the operation are written down; the author following the custom of his contemporaries. The two Tables *a* and *b* may illustrate the method adopted.

a. Multiplication.

ז ב א	127		127
ה ה ג	355		355
ה ג ג ב ג	32335	or	35
ה א א	115		35
א ו	61		21
ה ס	55		10
ה	45085		10
			6
ה ח ס ה ר			355
			45085

Ibn Ezra tests the correctness or at least the probable correctness of the result in the following way: consider each figure in the two factors and the product, or in the dividendus, divisor, quotient, and remainder, as units, find the sum of the figures in each quantity and divide the sums by nine; the remainder is called "balance" (Hebr. *moz'ne*); multiply the balances of the two factors, in case of multiplication, and divide by nine, if the balance is equal to that of the product, the result is assumed to be correct. In case of division, add to the product of the balances of divisor and quotient, the balance of the remainder, and divide the sum by nine, if the balance is equal to that of the dividendus the result is probably correct.

E. g. the test for the above multiplication sum: $1+2+7=10$ leaves the balance 1, $3+5+5$ leaves the balance 4, and $1\times4=4$; the balance of the product is likewise $4+5+8+5=22=2\times9+4$. The test for the division sum: divisor $9+9+9+9=36$ leaves 0; dividendus $9\times7=63$ leaves 0; quotient $7+7+7+8+5=34$ leaves 6; the remainder $5+5+6+2=18$ leaves 0; $0\times6+0=0$ and equal to the balance of the dividendus.

b. Division.

								ס	
							ס	א	
						א	ה	ה	
					ס	ח	ו	ו	
					א	ט	ג	ס	
					ז	ה	ה	ה	
				ס	ח	ח	ו	ו	
				א	ד	ט	ב	ג	ס
				ז	ז	ה	ה	ה	ו
			ס	ח	ח	ח	א	ב	ס
			א	ד	ד	ד	ד	ה	ב
Divid.			ז	ז	ז	ז	ז	ז	ז
Quotient						ז	ז	ח	ה
Divisor							ט	ט	ט
Remainder (the top figure of each column).								ה	ו

9999)777777777(77785

63

14
63

84
63

784
63

7784
63

14
63

85
63

791
63

7854
63

15
63

92
63

861
63

8554
72

13
72

63
72

562
72

5555
45

10
45

60
45

560
45

5562

The fifth chapter is devoted to fractions. The denominator (Hebr. *ha-moreh*) is best expressed by the product of simple numbers between two and nine, e.g. $\frac{1}{24}=\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$; $\frac{1}{60}=\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{5}$. A denominator that cannot be reduced to such simple, small factors, as e.g. all prime-numbers from 11 upwards, or numbers including such prime-numbers as factors, is called by Ibn Ezra "a number whose factors cannot be expressed by simple figures" (Hebr. שאין לו חלקים שיוכל האדם לבטא בהם[1]).

It is remarkable that Ibn Ezra finds also a common denominator for the multiplication of fractions; in the division of fractions it is intelligible; because through turning the fractions with different denominators into fractions with a common denominator, the problem is reduced to a division of whole numbers. Perhaps for the sake of uniformity the same is done in the case of multiplication. Another kind of fractions is mentioned by Ibn Ezra, as occurring in Astronomic calculations. They are similar to decimal fractions, with this difference, that instead of the different powers of 10 the denominators are formed by the different powers of 60. The following instance of multiplication may serve as an illustration:—

			ג	ד	ט	ב
			אא	דד	חא	ג
			בב	חח	וג	ו
		טט	וטג	בוא	זב	
	דד	וא	בז	בא		
גג	בנא	דה	ט			
גג	וא	טבג	טטר	בוב	גו	ו
		ב	ה	ח	ד	א
גג	וה	אג	דב	סג	ז	ז

					$3'''$	$4''$ $9'$ 2°
					11	44 18 3
					22	88 36 6
				99	396	162 27
			44	176	72	12
33	132	54	9			
33	176	329	499	212	63	6
		2	5	8	4	1
33	56	31	24	30	7	7

The last two chapters deal with proportions and square roots. Numerous problems are solved in illustration of the rules given. Some of the methods he marks as his own by introducing them by the word מצאתי "I have found"; e.g. the sum of the numbers from

[1] בהם refers to אחרים mentioned before or it is a slip for באחרים.

1 to n equals half the sum of n^2+n; the square of a number is equal to ten times the square of its third minus the square of its third.

$$a^2=(\frac{a}{3})^2 \cdot 10-(\frac{a}{3})^2.$$

The editor has done everything in his power to produce a correct text. Five different MSS. were consulted and collated and their differences duly and conscientiously registered. The notes likewise contain valuable information on the subject; but I do not see the editor's object in adding a literal German Translation. A short *résumé* of the book in terms intelligible to all familiar with modern Arithmetic would fully suffice. I am sure that the ordinary German reader will meet with far greater difficulty in his attempt to understand the German translation than the ordinary Hebrew reader will meet with when trying to master the original. Does any one understand: "Multiply minutes with minutes, and you obtain seconds as the product." The Hebrew is correct and intelligible: "Multiply two fractions whose denominator is 60 (ראשונים); the product is a fraction, with the denominator 60^2 (שניים)." The book without the translation is an excellent work.

M. FRIEDLÄNDER.

THE HAGGADAH ACCORDING TO THE RITE OF YEMEN.

The Haggadah according to the rite of Yemen, together with the Arabic-Hebrew Commentary, by WILLIAM H. GREENBURG. (London: David Nutt. 1896.)

DR. GREENBURG has rendered a service to Semitic philology and to Jewish Literature by editing the Yemen Haggadah together with the Arabic-Hebrew Commentary. The work is done carefully and conscientiously. A good many MSS. have been consulted and collated, and the variae lectiones are duly registered in the first set of footnotes. A second set of footnotes contains valuable philological remarks and references to Talmud and Midrash. The text of the Haggadah is on the whole the same as in the ordinary editions, with some interesting variations, of which the following are a few examples:—The first paragraph of our Haggadah begins כהא לחמא עניא; in the Yemen MSS. it is preceded by the words בבהילו יצאנו ממצרים